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Microstructure Effects on the Conjugate Heat Transfer Along a Vertical Circular Pin

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Nomenclature

B, Δ, λ	= dimensionless material parameters, $[B = R^2/j, \Delta = \kappa/\mu, \lambda = \gamma/(\mu j)]$
C_p	= specific heat
F	= dimensionless stream function
G	= dimensionless microrotation function
Gr	= Grashof number
h	= heat transfer coefficient
h	= dimensionless heat transfer coefficient
j	= microinertia per unit mass
k	= thermal conductivity
L	= length of the pin
N	= angular velocity
N_c	= convection-conduction parameter
Pr	= Prandtl number
Q	= overall heat transfer rate
q	= local heat flux
R	= radius of the pin
Re	= Reynolds number, (UR/ν)
r	= radial coordinate
T	= temperature
U_∞	= freestream velocity
u, v	= velocity components in x and r directions, respectively
x	= streamwise coordinate
α	= thermal diffusivity
η, ξ	= pseudosimilarity variables
θ	= dimensionless temperature
μ, κ, γ	= material constants
ρ	= density of fluid
σ	= surface curvature parameter, $(4L/R)\ Re^{-1/2}$
ψ	= stream function
Ω	= buoyancy parameter, (Gr/Re^2)

Subscripts

f	= properties of the fluid
w	= properties of the solid pin
0	= conditions of root of the pin
∞	= conditions at freestream

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Introduction

THE problem of conjugate convection and conduction for a vertical plate fin has been studied by Sparrow and Acharya[1] and Sparrow and Chyu.[2] Quite recently, Gorla[3] presented an analysis for the heat transfer characteristics of a laminar forced convective flow of a Newtonian fluid over a circular pin by the conjugate convection-conduction theory including radiative effects under optically thick limit approximation. The problem of conjugate natural convection from a vertical fin to micropolar fluids was studied by Lien and Chen.[4] The heat transfer characteristics of a moving cylinder in a quiescent micropolar fluid were studied by Gorla.[5] Moutsoglou[6] investigated the effects of the stretching of filaments on the cooling of fibers during the melt-spinning process.

In the present paper, consideration is given to a circular pin fin extending from a wall and transferring heat to a surrounding micropolar fluid. The temperature of the fin is not known a priori. The heat transfer coefficient along the circular pin is not prescribed but will be determined from a solution of the boundary-layer equations and its interaction with the pin conduction. Numerical results are presented for the local heat transfer coefficient and distribution of temperature along the length of the pin.

Analysis

Let us consider a uniform freestream flow of a micropolar fluid, with velocity U_∞ and temperature T_∞, approaching a circular pin of radius R and length L. The pin is attached to a wall of temperature T_0. The thermal conductivity of the pin is κ_w. The temperature of the pin, T_w, varies along its length. The axial and radial coordinates are taken to be x and r, respectively. The conservation equations, within Boussinesq approximation, are given in Ref. 5, and, therefore, will not be repeated to conserve space.

Preceding with the analysis, we now introduce the following variables:

$$\xi = \frac{x}{L}, \qquad \eta = \left(\frac{r^2 - R^2}{4RL}\right)\left(\frac{Re}{\xi}\right)^{1/2}$$

$$\psi(x,r) = R(\nu U_\infty x)^{1/2} F(\xi,\eta)$$

$$N(x,r) = \frac{R}{r}\left(\frac{U_\infty \nu}{R}\right)^{1/2} \frac{\rho}{4\kappa} G(\xi,\eta) \tag{1}$$

$$\theta = [T(x,r) - T_\infty]/[T_0 - T_\infty]$$

The transformed governing equations may be written as

$$(1 + \xi^{1/2}\sigma\eta)F''' + (F + \xi^{1/2}\sigma)F'' + \xi^{1/2}G' + 8\xi\Omega\theta$$
$$= 2\xi\left[F'\frac{\partial F'}{\partial \xi} - F''\frac{\partial F}{\partial \xi}\right] \tag{2}$$

$$\lambda(1 + \xi^{1/2}\sigma\eta)G'' + (2F + \xi^{1/2}\sigma)G' + \eta F'G$$
$$= \Delta\xi^{1/2}(1 + \xi^{1/2}\sigma\eta)ReF'' + \frac{8\Delta B\xi}{Re}G$$
$$+ \frac{4\xi^{1/2}}{Re^{1/2}}[1 + \xi^{1/2}\sigma\eta]^{-1}\left[FG + \xi G\frac{\partial F}{\partial \xi} - \frac{\eta}{2}GF'\right]$$
$$+ 2\xi\left[F' + \frac{\partial G}{\partial \xi} - G'\frac{\partial F}{\partial \xi}\right] \tag{3}$$

$$\frac{(1 + \xi^{1/2}\sigma\eta)\theta''}{Pr} + \left(F + \frac{\xi^{1/2}\sigma}{Pr}\right)\theta' = 2\xi\left[F'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial F}{\partial \xi}\right] \tag{4}$$

The transformed boundary conditions are given by

$$F(\xi,0) = F'(\xi,0) = 0, \quad G(\xi,0) = 0, \quad \theta(\xi,0) = \theta_w(\xi)$$
$$F'(\xi,\infty) = 1, \quad G(\xi,\infty) = 0, \quad \theta(\xi,\infty) = 0 \tag{5}$$

In the previous equations, primes denote partial differentiation with respect to η alone. Furthermore,

$$\sigma = (4L/R)(1/Re^{1/2}), \qquad \Omega = Gr/Re^2 \tag{6}$$

The coupling between the pin and the convective flow is expressed by the requirement that the pin and fluid temperatures and heat fluxes be continuous at the pin-fluid interface at all x locations. This requirement was expressed as

$$T_w = T_f \quad \text{and} \quad -K_f \frac{\partial T}{\partial r} = h\,(T_w - T_\infty)$$
$$\text{for } r = R \quad \text{and} \quad 0 \le x \le L \tag{7}$$

The pin conservation of energy equation is

$$\frac{d^2\theta_w}{d\xi^2} = N_c \bar{h} \theta_w \tag{8}$$

In Eq. (9),

$$\theta_w(\xi) = \frac{T_w(\xi) - T_\infty}{T_0 - T_\infty} \qquad N_c = \frac{Lk_f}{K_w R} Re^{1/2}$$

$$\bar{h} = -\frac{\theta'(\xi,0)}{\theta(\xi,0)} \xi^{1/2} \left(\frac{R}{L}\right)^{1/2} \tag{9}$$

The boundary conditions for Eq. (8) are

$$\xi = 0: \frac{d\theta_w}{d\xi} = 0, \qquad \xi = 1: \theta_w = 1 \tag{10}$$

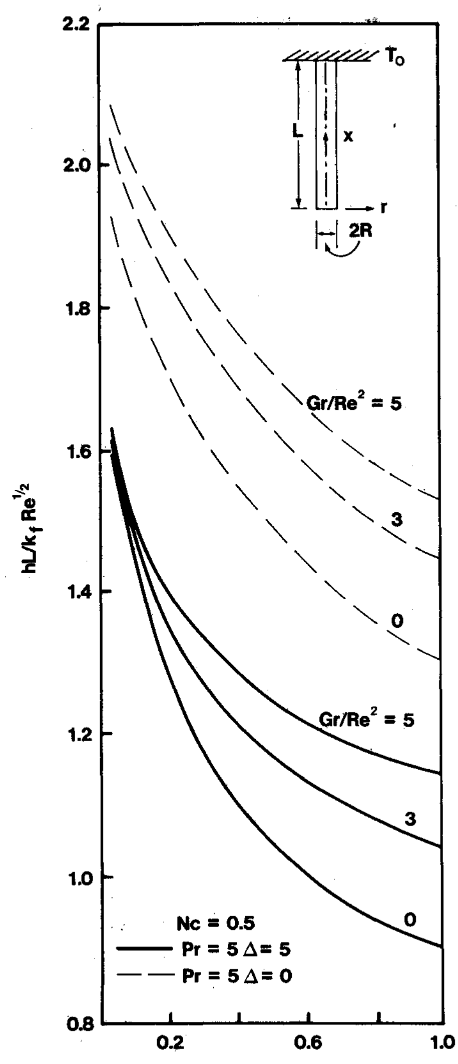

Fig. 1 Distribution of local heat-transfer coefficients along the pin surface.

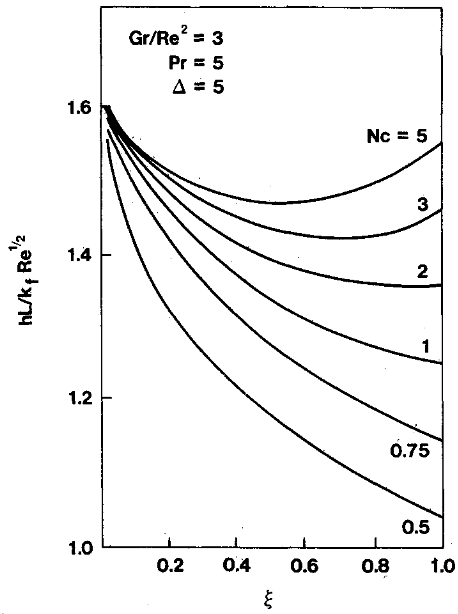

Fig. 2 Distribution of local heat flux along the pin surface.

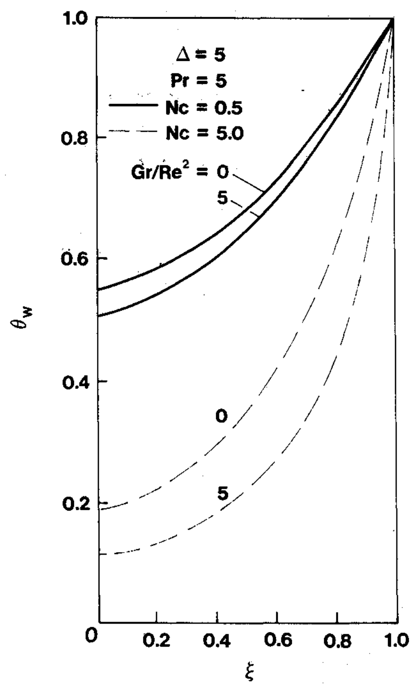

Fig. 3 Distribution of temperature of the pin.

Numerical Solution and Discussion of Results

The iterative procedure, as described by Gorla,[3] is used for the circular pin and the boundary layer. The boundary-layer equations were solved by an implicit finite-difference method described by Cebeci and Bradshaw.[7]

The dimensionless local heat flux may be written as

$$\frac{q_w L}{K_f (T_0 - T_\infty) Re^{1/2}} = -\frac{\theta'(\xi,0)}{2\xi^{1/2}} \tag{11}$$

Figures 1 and 2 display the numerical results for the local heat transfer coefficients at various axial locations of the pin. It may be observed that higher values of the conjugate convection-conduction parameter N_c represent higher local heat-transfer coefficients. As the buoyancy force parameter Ω increases, the magnitude of the heat transfer coefficients also increases. As the dimensionless material parameters Δ increases, the heat transfer coefficients decrease due to the presence of polymeric additives. We may note that $\Delta = 0$ denotes Newtonian fluids. The local heat transfer coefficients do not decrease monotonically in the flow direction for large values of N_c and Ω. They decrease first to some minimum value and then steadily increase with ξ. This is attributed to enhanced buoyancy associated with an increase in the wall to fluid temperature difference along the streamwise direction.

The results for the pin temperature distribution are illustrated in Fig. 3. As N_c or Ω increases, we see that the pin surface becomes more nonisothermal. In all cases, the pin temperature distributions decrease monotomically from root to tip.

Concluding Remarks

In this Note, we have presented an analysis for the conjugate convection and conduction heat transfer including buoyancy force on the forced flow of a micropolar fluid over a vertical circular pin. The overall heat transfer rate decreases with increasing values of the convection-conduction parameter N_c or decreasing values of the buoyancy parameter. The surface temperature variations of the pin from the root to tip increase with increasing values of N_c and the buoyancy force parameter Ω.

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Mixed Convection of Low Prandtl Number Fluid in the Annuli of Rotating Cylinders

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Nomenclature

$\hat{g}$	= gravitational vector
K_{eq}	= overall equivalent thermal conductivity
K_{eql}	= local equivalent thermal conductivity
L	= characteristics length, $(R_o - R_i)$
Pr	= Prandtl number, ν/α
R_i, R_o	= inner and outer cylinder radii
Ra	= Rayleigh number, $\beta g L^3 T_m/\alpha\nu$
Re	= Reynolds number, $R_i \omega L/\nu$
r	= radial coordinate
T_i, T_o	= temperatures of inner and outer cylinders
T_R, T_m	= reference temperature; $T_R = (T_i + T_o)/2$, $T_m = (T_i - T_o)/2$, respectively
t, T	= time and temperature, respectively
u, v	= radial and tangential velocity, respectively
α, β	= thermal diffusivity and coefficient of volumetric expansion
γ	= angle measured anticlockwise from the downward vertical through the center of the heater cylinder
Θ	= dimensionless temperature, $(T - T_R)/T_m$
ν	= kinematic viscosity
ρ	= reference density corresponding to T_R
ϕ	= angular coordinate
ψ, ζ	= stream function and vorticity, respectively
ω	= angular velocity of inner rotating cylinder

Introduction

CONVECTIVE fluid motion in a region bounded by two horizontal cylinders with parallel axes has been the subject of many studies in recent years. The mixed-convection problem in which buoyancy and centrifugal effects (created by heated rotating cylinders) is of practical concern in many technological applications, ranging from the control of chemical engineering process equipment, rotating machinery, and shafting, to the prediction of meteorological conditions. In machinery with the rotating shaft heated by electrical means through a mercury slip ring, the rotating shaft is immersed in mercury and the characteristics of the fluid flow and heat transfer processes are not well known. In most of the studies cited,[1-7] attention has been focused on fluids with Prandtl numbers of order one and larger, and none consider the rotating shaft immersed in mercury with a Prandtl number of 0.02. Liu et al.[1] measured overall heat transfer characteristics and temperature profiles for air, water, and silicone oil. Photographic studies of the flow patterns in air were first presented by Bishop and Carley.[2] Powe et al.[3] examined the critical Rayleigh number at which counter-rotating eddies begin to form for air. Kuehn and Goldstein[4] presented results of experimental and numerical studies of the motion of air and water within a horizontal annulus. More recent related studies were done by Fusegi et al.,[5] Bishop and Brandon,[6] and Bishop.[7] These studies, however, were concerned with fluids of $Pr \cong 1$ or higher.

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